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TAIL PROBABILITIES FOR A RISK PROCESS WITH SUBEXPONENTIAL JUMPS IN A REGENERATIVE AND DIFFUSION ENVIRONMENT

BY

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Abstract. In this paper we find a nonexponential Lundberg approximation of the ruin probability in a Cox model, in which a governing process has a regenerative structure and claims are light-tailed or have an intermediate regularly varying distribution. Examples include an intensity process being reflected Brownian motion, square functions of the Ornstein–Uhlenbeck process and splitting reflected Brownian bridges. In particular, we consider a non-Markovian intensity process.

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1. INTRODUCTION

This paper is concerned with a risk theory subject to a combination of two features: a stochastic modulation and regularly varying claim size distributions. We consider a canonical *surplus process* $\{S(t), t \geqslant 0\}$ given by

$$S(t) = \sum_{i=1}^{N(t)} U_i - t,$$

where $\{N(t), t \geqslant 0\}$ is a Cox process with an underlying cádlág process $\{X(t), t \geqslant 0\}$. That is, if a realization of the process $\{X(t), t \geqslant 0\}$ is $x(t) \in \mathcal{D}[0, +\infty)$, then for a nonnegative measurable function $\lambda\colon \boldsymbol{R} \to \boldsymbol{R}_+ \cup \{0\}$ the process $\{N(t), t \geqslant 0\}$ has the same law as a nonhomogeneous Poisson process $\{N^{(x)}(t), t \geqslant 0\}$ with an intensity function $\bar{\lambda}(t) = \lambda(x(t))$. The process $\{\lambda(X(t)), t \geqslant 0\}$ is called an *intensity process*. Thus *stochastic modulation* means that the surplus process is not time-homogeneous, but evolves in some random

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environment. A detailed discussion of Cox processes and their impact on risk theory is to be found in Grandell [19] and Rolski et al. [33]. The *claim sizes* $U_1, U_2, \ldots$ are i.i.d. r.v.'s independent of the process $\{N(t), t \geqslant 0\}$ with a common distribution function $F_U(x)$. Let u be an initial reserve and assume that $S(t) \to -\infty$ a.e. as $t \to +\infty$. An *infinite horizon ruin probability* is then

$$\psi(u) = \boldsymbol{P}\big(\sup_{t \geqslant 0} S(t) > u\big). \tag{1.1}$$

The model, in which $\{N(t), t \geqslant 0\}$ is Coxian, is called the *Björk–Grandell model* which goes to the pioneering paper Björk and Grandell [10]. In that paper one derives by a martingale approach an exponential upper bound of $\psi(u)$ when an intensity process has piecewise constant realizations and claim sizes are light-tailed. Further generalizations can be found in Embrechts et al. [16] (finite time non-Markovian intensities) and Grigelionis [22]. In applications one needs also to consider environment which changes more widely like it is in the case of diffusions or Gaussian processes. Grandell and Schmidli [21] and Palmowski [29] find a Lundberg upper bound and a Lundberg approximation of $\psi(u)$ when an intensity process is governed by a diffusion process and claim sizes are light-tailed. These papers fail to capture another main feature considered in this paper, namely, that of regularly varying tails. Relevance of heavy-tail conditions can be found e.g. in Embrechts and Veraverbeke [17] and Klüppelberg [26]. Asmussen et al. [5] find the nonexponential asymptotics in the Björk–Grandell model when the governing process is a finite-state Markov process and the claim size has a heavy-tailed distribution. Asmussen et al. [8] generalize it to the case when $\{S(t), t \geqslant 0\}$ has a regenerative structure. In this paper we apply this result to get the asymptotics of $\psi(u)$ when a rate of arrival of a claim at time t is a function $\lambda(x)$ of a regenerative process $\{X(t), t \geqslant 0\}$; in particular, when $\{X(t), t \geqslant 0\}$ is a recurrent diffusion process.

Denote by $0 = T_0 \leqslant T_1 \leqslant T_2 \leqslant T_3 \leqslant \ldots$ the regenerative epochs of the regenerative process $\{X(t), t \geqslant 0\}$. That is, $\{T_{n+1} - T_n\}$, $n = 0, 1, \ldots$, is a sequence of i.i.d. r.v.'s with generic interregenerative time T. We say that the r.v. G is *heavier* than the r.v. H if

$$\limsup_{x \to \infty} \boldsymbol{P}(H > x)/\boldsymbol{P}(G > x) < +\infty.$$

Define the r.v.

$$Z = \int_{T_n}^{T_{n+1}} \lambda(X(t))\, dt.$$

Denote by $F(x)$ the heavier distribution from distributions of variables Z and U. Further on, we will assume that $F(x)$ has a regularly varying distribution. We will write $f(x) \sim g(x)$ as $x \to +\infty$ if $\lim_{x \to +\infty} f(x)/g(x) = 1$. In this paper we show that if $ET < +\infty$, then under some mild assumptions

$$\psi(u) \sim CF^s(u) \quad \text{as } u \to +\infty,$$

where F^s is the residual distribution of F and the constant C is given explicitly (see Theorem 3.2 (i)–(iii)). Thus even in the case of light-tailed claims one can get the nonexponential asymptotics. The asymptotics of $\psi(u)$ in this case depends on the distribution of the interarrival time T only via its mean if $ET < +\infty$. If $ET = +\infty$, then its tail also has an impact on the rate of the asymptotics of the ruin probability (see Theorem 3.2 (iv) and Section 5.2). The method of proof of the main Theorem 3.2 is based on the Karamata–Tauberian Theorem and the Kingman–Taylor expansion of the Laplace transform (see Stam [35], Cohen [15] and Asmussen et al. [8], Corollaries 3.1 and 3.2).

To apply this result for the specific governing process $\{X(t),\, t \geqslant 0\}$ one has to determine the asymptotic tails of r.v.'s $Z = \int_{T_n}^{T_{n+1}} \lambda(X(t))\,dt$ and T and their means. We refer to Asmussen et al. [7] for similar functionals. Note that the r.v. Z may be heavy-tailed (Section 4) or light-tailed (Section 5). In the second part of this paper we calculate some examples presenting there main techniques useful in solving this problem.

To prove that the r.v. Z is light-tailed we generalize Wentzell [38], p. 265, in the following way. Consider a family of diffusion processes $\{X_w(t),\, t \geqslant 0\}$ parametrized by $w \geqslant 0$ starting at $X_w(0) = x_w$. Let τ_w be an exit time from a compact set D. If there exists $w_0 > 0$ such that $E_{x_w}^{X_w} \tau_w$ is uniformly bounded for all $0 < w \leqslant w_0$, then $E_{x_w}^{X_w} \exp\{w\tau_w\}$ is also uniformly bounded.

We calculate the asymptotics of the tail of the distribution of Z and its mean using the Laplace transform method. In most cases we take the square function $\lambda(x)$. Then the method of computing the m.g.f. (or the Laplace transform) consists in changing probability so that the quadratic functional disappears and the remaining problem is to calculate the m.g.f. and the Laplace transform of some hitting or exit times. In other words, we linearize the original problem by transferring the computational problem for a variable belonging to a second Wiener chaos to computations for a variable in the first chaos. One can calculate the Laplace transform of hitting and exit times using the Feynman–Kac formula (see Itô and McKean [23], Wentzell [38] and Borodin and Salminen [12]).

The rest of the paper is organized as follows. In Section 2 we recall the Karamata–Tauberian theorem. The main Theorem 3.2 is stated in Section 3. We consider the following examples of the governing process $\{X(t),\, t \geqslant 0\}$ and the function $\lambda(x)$: the semi-Markov process and $\lambda(x) = x$ (Section 4), the reflected Brownian motion at 0 and 1 and $\lambda(x) = x$ (Section 5.1), the Brownian motion and $\lambda(x) = e^{-\gamma|x|}$ (Section 5.2), the Ornstein–Uhlenbeck process and $\lambda(x) = x^2 + k$ (Section 5.3) and $\lambda(x) = (x+p)^2$ (Section 5.4), and finally the splitting Brownian bridges and $\lambda(x) = |x|$ (Section 5.5).

2. PRELIMINARIES

The main technique useful in finding the asymptotics of $\psi(u)$ is the Karamata–Tauberian Theorem, which we recall now. The critical index is defined in extended real numbers by

$$\alpha_K = \inf\{v\colon E|K|^v = +\infty\}. \tag{2.1}$$

That is, if there exists $\delta > 0$ such that $Ee^{\delta K} < +\infty$, then $\alpha_K = +\infty$. We say that the r.v. K has a *regularly varying distribution* if

$$P(K > x) \sim x^{-\alpha_K} l_K(x) \quad \text{as } x \to +\infty$$

for a slowly varying function $l_K(x)$. Let us put

$$m_{i,K} = E|K|^i.$$

The Karamata–Tauberian Theorem relates the tail behaviour of a distribution function to the asymptotic behaviour of its Laplace transform at the origin. For a variable K let $\alpha_K < +\infty$ and define $n = [\alpha_K]$. Then by Kingman and Taylor [25] the Laplace transform $\tilde{F}^K(s)$ of the r.v. K may be expanded in the Taylor series as far as the s^n term:

$$\tilde{F}^K(s) = \sum_{k=0}^{n} m_{k,K}(-s)^k/k! + o(s^n) \quad \text{as } s \to 0.$$

Let

$$f_n^K(s) = (-1)^n \Big(\tilde{F}^K(s) - \sum_{k=0}^{n} m_{k,K}(-s)^k/k!\Big).$$

We will write $f(x) \sim g(x)$ as $x \to 0$ if $\lim_{x\to 0} f(x)/g(x) = 1$. From Bingham et al. [9], p. 333, we have the following theorem.

THEOREM 2.1. *Let $l_K(x)$ be a slowly varying function. Then the following are equivalent*:

$$f_n^K(s) \sim s^{\alpha_K} l_K(1/s) \quad \text{as } s \to 0, \tag{2.2}$$

$$P(K > x) \sim \frac{(-1)^n}{\Gamma(1-\alpha_K)} x^{-\alpha_K} l_K(x) \quad \text{as } x \to +\infty. \tag{2.3}$$

From Feller [18], Theorem 2, p. 445, we have the following theorem.

THEOREM 2.2. *Consider some function $L(x)$ and $\alpha > 0$. Let us define*

$$\tilde{F}^L(s) = \int_0^\infty e^{-sx}\, dL(x).$$

Then for a slowly varying function $l_L(x)$ *the following are equivalent*:

(2.4) $$\tilde{F}^L(s) \sim s^{-\alpha} l_L(1/s) \quad \text{as } s \to 0,$$

(2.5) $$L(x) \sim \frac{1}{\Gamma(1+\alpha)} x^{\alpha} l_L(x) \quad \text{as } x \to +\infty.$$

3. MAIN THEOREM

Let $\{\lambda(X(t)), t \geqslant 0\}$ be an intensity process defined by the regenerative process $\{X(t), t \geqslant 0\}$ and a nonnegative function $\lambda(x)$. Then the surplus process $\{S(t), t \geqslant 0\}$ also has a regenerative structure. We let S be the increment of $\{S(t), t \geqslant 0\}$ during the generic cycle T, that is

$$S = \sum_{i=1}^{N(T)} U_i - T.$$

Further, let

$$S^+ = \sum_{i=1}^{N(T)} U_i \quad \text{and} \quad Z = \int_0^T \lambda(X(s))\, ds.$$

CONDITION A. We assume that

(A) $$P(S^+ > x) \sim P(S > x) \sim x^{-\alpha_S} l_S(x),$$

where $\alpha_S > 0$ and $l_S(x)$ is a slowly varying function.

By Asmussen et al. [8], Lemma 5.1, we have the following lemma.

LEMMA 3.1. *Assume that the following condition holds*:

CONDITION B.

(B) $$\exists \delta > 0: \; Ee^{\delta Z} < +\infty.$$

Then condition (A) *holds.*

Note that

$$S(T_n) + \sum_{i=N(T_n)+1}^{N(T_{n+1})} U_i - (T_{n+1} - T_n) \leqslant \sup_{T_n \leqslant t < T_{n+1}} S(t) \leqslant S(T_n) + \sum_{i=N(T_n)+1}^{N(T_{n+1})} U_i.$$

Thus under (A), following Asmussen and Klüppelberg [6] and Asmussen et al. [8], Theorem 3.3, we have

$$\psi(u) \sim P\big(\max_{n \geqslant 1} (Y_1 + Y_2 + \ldots + Y_n) > u\big),$$

where Y_n are i.i.d. r.v.'s such that $Y_n \stackrel{D}{=} S$. Note also that if $\nu = E|S| < +\infty$, then $\alpha_S > 1$.

THEOREM 3.1. *Assume that* (A) *holds.*
(i) *If* $\nu < +\infty$ *and* $ES < 0$, *then*

$$\psi(u) \sim \frac{1}{\alpha_S - 1} \frac{1}{\nu} l_S(u) u^{-\alpha_S + 1}. \tag{3.1}$$

(ii) *If* $\nu = +\infty$, $l_S(x) = c_1$ *and*

$$P(T > x) \sim c_2 x^{-\beta} \tag{3.2}$$

for $0 < \beta < 1$ *and* $\beta < \alpha_S$, *then*

$$\psi(u) \sim \frac{\sin(\beta\pi)}{\beta\pi} \frac{c_1}{c_2} u^{\beta - \alpha_S} \int_0^{+\infty} y^{\beta-1}(1+y)^{-\alpha_S} dy. \tag{3.3}$$

Proof. Part (i) follows from Corollary 3.1 of Asmussen et al. [8].

We now prove (ii). Denote by $G^+(x)$ and $G^-(x)$ the ascending and descending weak ladder height distributions, respectively, of random walk $Y_1 + Y_2 + \ldots + Y_n$. Thus $G^+(x)$ and $G^-(x)$ are concentrated on $[0, +\infty)$ and $(-\infty, 0]$, respectively. From the Wiener–Hopf factorization (see Borovkov [13], (33), p. 165) we have

$$G^-(-x) \sim \frac{c_2}{1-p} x^{-\beta} \quad \text{as } x \to +\infty, \tag{3.4}$$

where $p = \psi(0)$. Let

$$H^-(t) = \sum_{k=0}^{+\infty} (G^-)^{*k}(-t), \quad t \geqslant 0,$$

and $\tilde{F}^{G^-}(s) = \int_0^{+\infty} e^{-sx} dG^-(-x)$. Then the Laplace transform of H^- is equal to

$$\tilde{F}^{H^-}(s) = \frac{1}{1 - \tilde{F}^{G^-}(s)}.$$

From the Karamata–Tauberian Theorem 2.1 the following holds:

$$\lim_{s \to 0} \frac{\tilde{F}^{H^-}(s)}{s^{-\beta}} = \lim_{s \to 0} \frac{s^\beta}{1 - \tilde{F}^{G^-}(s)} = \frac{1-p}{\Gamma(1-\beta) c_2}.$$

Thus, by the Karamata–Tauberian Theorem 2.2,

$$H^-(t) \sim \frac{1-p}{\Gamma(1-\beta)\Gamma(1+\beta) c_2} t^\beta = \frac{(1-p)\sin(\beta\pi)}{\beta\pi c_2} t^\beta,$$

which completes the proof of (ii) in view of Borovkov [13], p. 180, and Lemma 2, p. 173. ∎

Note that the Laplace transform of S^+ is equal to

$$\tilde{F}^{S^+}(s) = Ee^{-sS^+} = E^X (E^U e^{-sU})^{N(T_{n+1}) - N(T_n)}$$
$$= E^X \exp\{-\log((E^U e^{-sU})^{-1}) \int_0^T \lambda(X(t)) dt\},$$

where E^X and E^U are expectations with respect to the law of the process $\{X(t), t \geqslant 0\}$ and the r.v. U. That is,

$$\tilde{F}^{S^+}(s) = \tilde{F}^Z(\log \tilde{F}^U(s)^{-1}). \tag{3.5}$$

Moreover, if $\nu < +\infty$, then $\nu = ET - EZEU = m_{1,T} - m_{1,Z} m_{1,U}$. Thus, if the following condition is satisfied:

CONDITION S.

$$\nu < +\infty \quad \text{and} \quad m_{1,T} > m_{1,U} m_{1,Z}, \tag{S}$$

then a stability condition $S(t) \to -\infty$ a.e. as $t \to +\infty$ holds.

We assume that the heavier random variable out of variables U and Z has a regularly varying distribution. Thus

$$\alpha_{Z,U} = \min\{\alpha_U, \alpha_Z\} < +\infty,$$

and if $\alpha_Z < \alpha_U$ ($\alpha_U < \alpha_Z$), then Z (U) is heavier than U (Z). In particular, if U or Z has a regularly varying distribution, then one of the following two conditions holds:

CONDITION U.

$$P(U > x) \sim l_U(x) x^{-\alpha_U}, \tag{U}$$

CONDITION Z.

$$P(Z > x) \sim l_Z(x) x^{-\alpha_Z} \tag{Z}$$

for slowly varying functions $l_U(x)$ and $l_Z(x)$, respectively. Using the Karamata–Tauberian Theorem we can prove the following theorem (see also Asmussen et al. [8], Schmidli [34], Stam [35] and Grandell [20] for related results).

THEOREM 3.2. *Assume that condition* (A) *and at least one of the conditions* (Z) *or* (U) *holds.*

(i) *If* $1 < \alpha_Z < \alpha_U$ *and* (Z), (S) *hold, then*

$$\psi(u) \sim C_1 l_Z(u) u^{-\alpha_Z+1}, \tag{3.6}$$

where

$$C_1 = \frac{1}{\alpha_Z - 1} m_{1,U}^{\alpha_Z} \frac{1}{m_{1,T} - m_{1,U} m_{1,Z}}. \tag{3.7}$$

(ii) *If* $1 < \alpha_U < \alpha_Z$ *and* (U), (S) *hold, then*

$$\psi(u) \sim C_2 l_U(u) u^{-\alpha_U+1}, \tag{3.8}$$

where

$$C_2 = \frac{1}{\alpha_U - 1} m_{1,Z} \frac{1}{m_{1,T} - m_{1,U} m_{1,Z}}. \tag{3.9}$$

(iii) *If* $\alpha_Z = \alpha_U$, $l_Z(x) = l_U(x)$ *and* (U), (Z), (S) *hold, then*

$$\psi(u) \sim C_3 l_U(u) u^{-\alpha_U+1}, \tag{3.10}$$

where $C_3 = C_1 + C_2$.

(iv) *Assume that* T *fulfills* (3.2) *for* $0 < \beta < 1$. *Then*

$$\psi(u) \sim C_4 u^{\beta - \alpha_{Z,U}}, \tag{3.11}$$

where

$$C_4 = \frac{\sin(\beta\pi)}{\beta\pi} \frac{m_{1,U}^{\alpha_Z} c_1}{c_2} \int_0^{+\infty} y^{\beta-1}(1+y)^{-\alpha_Z} dy \tag{3.12}$$

when (Z) *holds and* $l_Z(x) = c_1$, $\beta < \alpha_Z < \alpha_U$; *and*

$$C_4 = \frac{\sin(\beta\pi)}{\beta\pi} \frac{m_{1,Z} c_1}{c_2} \int_0^{+\infty} y^{\beta-1}(1+y)^{-\alpha_U} dy \tag{3.13}$$

when (U) *holds and* $l_U(x) = c_1$, $\beta < \alpha_U < \alpha_Z$.

Proof. We prove (i). The statements (ii)–(iv) can be proved in a very similar way. To prove (i), by Theorem 3.1 it suffices to show that

$$P(S^+ > x) \sim m_{1,U}^{\alpha_Z} l_Z(x) x^{-\alpha_Z}. \tag{3.14}$$

Let $k = [\alpha_Z]$ and $l = [\alpha_U]$ if $\alpha_U < +\infty$ and take any $l > k$ if $\alpha_U = +\infty$. We will write $g(s) = O_1(f(s))$ if $\lim_{s\to 0} g(s)/f(s) = 1$. At the beginning we consider the case when $l > k$. By the Karamata–Tauberian Theorem 2.1 we have

$$\tilde{F}^Z(s) = 1 - m_{1,Z} s + \frac{1}{2} m_{2,Z} s^2 - \ldots + \frac{(-1)^k}{k!} m_{k,Z} s^k + O_1\big((-1)^k \Gamma(1-\alpha_Z) s^{\alpha_Z} l_Z(1/s)\big), \tag{3.15}$$

and by the Kingman–Taylor expansion of the Laplace transform we obtain

$$\tilde{F}^U(s) = 1 - m_{1,U} s + \frac{1}{2} m_{2,U} s^2 - \ldots + \frac{(-1)^l}{l!} m_{l,U} s^l + o(s^l). \tag{3.16}$$

Hence by (3.5) we have

$$\begin{aligned} \tilde{F}^{S^+}(s) = 1 &- m_{1,Z} \log\big(\tilde{F}^U(s)^{-1}\big) + \frac{1}{2} m_{2,Z} \big(\log\big(\tilde{F}^U(s)^{-1}\big)\big)^2 - \ldots \\ &+ \frac{(-1)^k}{k!} \big(\log\big(\tilde{F}^U(s)^{-1}\big)\big)^k \\ &+ O_1\Big[(-1)^k \Gamma(1-\alpha_Z) \big(\log\big(\tilde{F}^U(s)^{-1}\big)\big)^{\alpha_Z} l_Z\big(1/\log\big(\tilde{F}^U(s)^{-1}\big)\big)\Big]. \end{aligned} \tag{3.17}$$

Note that for $x > 0$ such that $|x-1| < 1$

$$\log(1/x) = \sum_{i=1}^{+\infty} \frac{(-1)^i}{i}(x-1)^i.$$

Hence

$$\log(\tilde{F}^U(s)^{-1}) = \sum_{i=1}^{+\infty} \frac{(-1)^i}{i}(\tilde{F}^U(s)-1)^i.$$

Consequently, (3.15)–(3.17) under the assumption $k < l$ imply

$$\begin{aligned}\tilde{F}^{S^+}(s) &= 1-m_{1,Z}m_{1,U}s+\tfrac{1}{2}(m_{2,Z}m_{1,U}^2-m_{1,Z}m_{2,U})s^2-\ldots+m_{k,S^+}s^k \\ &\quad +O_1\left[(-1)^k\Gamma(1-\alpha_Z)(\tilde{F}^U(s)-1)^{\alpha_Z}l_Z\big(1/(\tilde{F}^U(s)-1)\big)\right] \\ &= 1-m_{1,Z}m_{1,U}s+\ldots+m_{k,S^+}s^k+O_1\left[(-1)^k\Gamma(1-\alpha_Z)m_{1,U}^{\alpha_Z}s^{\alpha_Z}l_Z\big(1/(m_{1,U}s)\big)\right] \\ &= 1-m_{1,Z}m_{1,U}s+\ldots+m_{k,S^+}s^k+O_1\left[(-1)^k\Gamma(1-\alpha_Z)m_{1,U}^{\alpha_Z}s^{\alpha_Z}l_Z(1/s)\right].\end{aligned}$$

Thus

$$f_k^{S^+}(s) \sim (-1)^k\Gamma(1-\alpha_Z)m_{1,U}^{\alpha_Z}s^{\alpha_Z}l_Z(1/s) \quad \text{as } s\to 0,$$

which completes the proof in the first case in view of (3.14) and the Karamata–Tauberian Theorem 2.1. If $k = l$, by Thorisson [37], Theorem 3.1, one can consider two modified risk models in which claim sizes $U^{(1)}$ and $U^{(2)}$ have regularly varying distributions and fulfill $U^{(1)} \leqslant U \leqslant U^{(2)}$ with $\alpha_Z < \alpha_{U^{(2)}} \leqslant \alpha_U \leqslant \alpha_{U^{(1)}}$, $m_{1,U^{(2)}} = m_{1,U}+\varepsilon$, $m_{1,U^{(1)}} = m_{1,U}-\varepsilon$ for $\varepsilon > 0$ and $l = [\alpha_{U^{(1)}}] = [\alpha_U] = [\alpha_{U^{(2)}}]$. Then, by Thorisson [37], Theorem 3.1,

$$P(S^{+,(1)} > x) \leqslant P(S^+ > x) \leqslant P(S^{+,(2)} > x),$$

where $S^{+,(1)}$ and $S^{+,(2)}$ are r.v. S^+ defined in modified models. Note that asymptotics in (3.14) depends on U only through its mean. Thus letting $\varepsilon \to 0$ we prove (3.14) in general if we show it assuming that the r.v. U has a regularly varying distribution. Under this assumption the assertion of (i) follows by similar considerations to those in the previous case by taking

$$\tilde{F}^U(s) = 1-m_{1,U}s+\frac{1}{2}m_{2,U}s^2-\ldots+\frac{(-1)^l}{l!}m_{l,U}s^l + O_1\big((-1)^l\Gamma(1-\alpha_U)s^{\alpha_U}l_U(1/s)\big) \tag{3.18}$$

instead of (3.16).

Remark 3.1. Similar results can be also obtained in the so-called delayed case, when $T_0 > 0$. Let us put $S_0^+ = \sum_{i=1}^{N(T_0)} U_i$ and $Z_0 = \int_0^{T_0} \lambda(X(t))\,dt$. If $P(S_0^+ > x) = o(l_S(x)x^{-\alpha_S+1})$, then the ruin probability $\psi_0(u)$ in the delayed case is asymptotically equivalent to the ruin probability $\psi(u)$ in the so-called zero-delayed case (when $T_0 = 0$). That is,

$$\psi_0(u) \sim \psi(u) \quad \text{as } u \to +\infty.$$

This is the case when the claim size U has the regularly varying distribution given in condition (U) and there exists a $\delta > 0$ such that $E\exp\{\delta Z_0\} < +\infty$. See Asmussen et al. [8], Corollary 3.2, for other relations between $\psi(u)$ and $\psi_0(u)$.

COROLLARY 3.1. *If conditions* (B), (U) *and* (S) *are fulfilled, then*

$$\psi(u) \sim C_2 l_U(u) u^{-\alpha_U+1}, \tag{3.19}$$

where C_2 *is given in* (3.9).

4. SEMI-MARKOV MODEL

Let $\{T_n\}_{n=1}^{+\infty}$ be the renewal process. That is, $T_{n+1} - T_n$ are i.i.d. r.v.'s. On the time interval $[T_n, T_{n+1})$ the process $\{X(t), t \geqslant 0\}$ is equal to a positive r.v. Δ_n. Random variables $\{\Delta_n\}_{n=1}^{+\infty}$ are i.i.d. and independent of $\{T_n\}_{n=1}^{+\infty}$. Moreover, let $\lambda(x) = x$. Thus $Z = T\Delta$, where Δ is a generic r.v. Δ_n. We can change the distributions of T and Δ in such a way that we can get all possible cases (i)–(iv) in the main Theorem 3.2 (see Grandell [20], Schmidli [34]). In particular, we can consider the Ammeter [3] model when $T = 1$. Then obviously condition (A) holds. From Theorem 3.2 (i) we obtain the following theorem.

THEOREM 4.1. *Assume that there exists a* $\delta > 0$ *such that* $E\exp\{\delta U\} < +\infty$ *and*

$$P(\Delta > x) \sim x^{-\alpha_\Delta} l_\Delta(x) \quad \text{as } x \to +\infty$$

for the slowly varying function $l_\Delta(x)$ *and* $\alpha_\Delta > 1$. *If* $m_{1,U} m_{1,\Delta} < 1$, *then we have the following asymptotics*:

$$\psi(u) \sim \frac{1}{\alpha_\Delta - 1} m_{1,U}^{\alpha_\Delta} \frac{1}{1 - m_{1,U} m_{1,\Delta}} l_\Delta(u) u^{-\alpha_\Delta + 1}.$$

Hence one can get the regularly varying asymptotics of the ruin probability $\psi(u)$ even when the claim sizes U are light-tailed.

5. DIFFUSION PROCESSES

On a probability space $(\mathscr{C}[0, +\infty), \mathscr{F}, \{\mathscr{F}_t^X\}_{t \geqslant 0}, P^X)$ let us consider a canonical diffusion process $\{X(t), t \geqslant 0\}$, where $\{\mathscr{F}_t^X\}_{t \geqslant 0}$ is a natural filtration and $\mathscr{F} = \bigvee_{t \geqslant 0} \mathscr{F}_t^X$. The process $\{X(t), t \geqslant 0\}$ has the following extended generator:

$$(Af)(x) = \frac{1}{2} a(x) \frac{d^2}{dx^2} f(x) + b(x) \frac{d}{dx} f(x)$$

for $f \in \mathscr{C}^2(\boldsymbol{R})$. Assume that there exists a constant L such that

$$a^2(x)+|b(x)| \leqslant L(1+|x|) \tag{5.1}$$

and that there exists, for each constant $C > 0$, a constant L_C such that

$$|a^2(x)-a^2(y)|+|b(x)-b(y)| \leqslant L_C|x-y| \quad \text{for } |x| \leqslant C \text{ and } |y| \leqslant C. \tag{5.2}$$

Further on, we will consider only the recurrent diffusion process $\{X(t),\, t \geqslant 0\}$. That is, any possible state is reached from any other state with probability 1. Let $X(0) = 0$ and $T_0 = 0$. In this paper we consider two kinds of regeneration moments: $T_n = n$ $(n \in N)$ and

$$T_{n+1} = \inf\{t \geqslant S_n\colon\, X(t) = 0\}, \tag{5.3}$$

where

$$S_n = \inf\{t \geqslant T_n\colon\, |X(t)| = 1\}, \qquad n = 0, 1, 2, \ldots \tag{5.4}$$

In this case $Z = \int_0^T \lambda(X(s))\,ds$, where $T = T_1$.

5.1. Reflected Brownian motion and $\lambda(x) = x$. Assume that the claim size U has the regularly varying distribution given in condition (U). Let $\{B(t),\, t \geqslant 0\}$ be a Brownian motion starting at $B(0) = 0$. Set $s(y) = (-1)^{[y]}$ and

$$S(x) = \int_0^x s(y)\,dy.$$

Thus $S(x)$ is a "saw-tooth" function with $S(x) = |x|$ for $-1 \leqslant x \leqslant 1$ and with a period 2. Assume that $\lambda(x) = x$. Then the intensity process $\{X(t) = S(B(t)),\, t \geqslant 0\}$ is a reflected Brownian motion with boundaries 0 and 1. The regeneration moments are defined by (5.3). Note that

$$E_0^X e^{\delta T} = E_1^B \exp\{\delta T'\}\, E_0^B \exp\{\delta S_0\} = (E_0^B \exp\{\delta S_0\})^2, \tag{5.5}$$

where

$$T' = \inf\{t \geqslant 0\colon\, |B(t)-1| = 1\}$$

and E_x^B is the expectation with respect to P^B when the Brownian motion $\{B(t),\, t \geqslant 0\}$ starts at x. The time S_0 is defined in (5.4). By Wentzell [38], p. 259, we have

$$E_0^B S_0 = 1. \tag{5.6}$$

Thus

$$m_{1,T} = 2E_0^B S_0 = 2. \tag{5.7}$$

Moreover, by Wentzell [38], p. 265, we have the following lemma.

LEMMA 5.1. *If τ is an exit time by a diffusion process from a compact set D and $E\tau \leqslant M < +\infty$, then*

$$Ee^{\delta\tau} \leqslant 1+\frac{\delta}{1-\delta M}E\tau \leqslant 1+\frac{\delta M}{1-\delta M} \quad \textit{for } 0 \leqslant \delta < M^{-1}.$$

Thus, by Lemma 5.1 and (5.5), (5.6), there exists a $\delta > 0$ such that

$$E_0^X e^{\delta T} < +\infty. \tag{5.8}$$

Note also that $0 \leqslant X(t) \leqslant 1$, and hence condition (B) is fulfilled:

$$E_0^X e^{\delta Z} = E_0^X \exp\{\delta \int_0^T X(t)\,dt\} \leqslant E_0^X e^{\delta T} < +\infty. \tag{5.9}$$

If (S) and (U) hold, then from Corollary 3.1 we obtain

$$\psi(u) \sim C_2\, l_U(u)\, u^{-\alpha_U+1},$$

where C_2 is given in (3.9). To calculate C_2 explicitly we find by the Markov property and the symmetry of the Brownian motion that

$$\begin{aligned} m_{1,Z} &= E_0^X \int_0^T X(s)\,ds = E_0^B \int_0^{S_0} |B(t)|\,dt + E_1^B \int_0^{T'} \big(1-|1-B(t)|\big)\,dt \\ &= E_0^B \int_0^{S_0} |B(t)|\,dt + E_1^B T' - E_0^B \int_0^{S_0} |B(t)|\,dt = 1. \end{aligned}$$

Summarizing we have the following theorem.

THEOREM 5.1. *Assume that the claim size U has the regularly varying distribution* (U) *with $\alpha_U > 1$ and $EU < 2$. Moreover, let the intensity process $\{X(t),\, t \geqslant 0\}$ be the reflecting Brownian motion reflecting at barriers* 0 *and* 1. *Then*

$$\psi(u) \sim \frac{1}{\alpha_U - 1}\,\frac{1}{2-m_{1,U}}\, l_U(u)\, u^{-\alpha_U+1}.$$

5.2. Brownian motion and $\lambda(x) = e^{-\gamma|x|}$. Assume that the claim size U has the regularly varying distribution (U) with index $\alpha_U > \frac{1}{2}$ and $l_U(x) = c_1$ for some constant c_1. Let the governing process $\{X(t) = B(t),\, t \geqslant 0\}$ be the Brownian motion starting at $B(0) = 0$ and $\lambda(x) = e^{-\gamma|x|}$. That is, $\{\exp(-\gamma|B(t)|),\, t \geqslant 0\}$ is the intensity Markov process. The regeneration moments are defined by (5.3). Then, by symmetry and the Markov property of the Brownian motion, we have

$$T \stackrel{D}{=} S_0 + \hat{T},$$

where

$$\hat{T} = \inf\{t \geqslant 0\colon\ B(t) = 0,\ B(0) = 1\}$$

and S_0 is defined in (5.4). Note that $E_0^B S_0 = 1$ and $E_1^B \hat{T} = +\infty$. Hence

$$E_0^B T = +\infty. \tag{5.10}$$

Moreover, by Karatzas and Shreve [24], p. 96,

$$P(T > t) \sim \frac{1}{\sqrt{\pi}} t^{-1/2}. \tag{5.11}$$

Thus $\alpha_T = \frac{1}{2}$. Note that

$$E_0^B e^{\delta Z} = E_1^B \exp\{\delta Z_1\} E_0^B \exp\{\delta Z_2\},$$

where $Z_1 = \int_0^{\hat{T}} e^{-\gamma|B(t)|} dt$ and $Z_2 = \int_0^{S_0} e^{-\gamma|B(t)|} dt$. Moreover, by (5.6) and Lemma 5.1,

$$E_0^B \exp\{\delta Z_2\} \leqslant E_0^B \exp\{\delta S_0\} < +\infty \tag{5.12}$$

for some $\delta > 0$. Let $T(R) = \inf\{t \geqslant 0\colon B(t) = R, B(0) = 1\}$. Then, by the Monotone Convergence Theorem,

$$E_1^B \exp\{\delta Z_1\} = \lim_{R \to +\infty} E_1^B \exp\{\delta \int_0^{\hat{T} \wedge T(R)} e^{-\gamma|B(t)|} dt\}.$$

Thus from the Feynman–Kac formula (see also Chung and Zhao [14], Theorem 9.22) we infer that for sufficiently small $\delta > 0$ the following holds:

$$E_1^B \exp\{\delta Z_1\} = \frac{J_0\left(2\left(\sqrt{2\delta}/\gamma \sqrt{\exp\{\gamma\}}\right)\right)}{J_0\left(2\left(\sqrt{2\delta}/\gamma\right)\right)} < +\infty, \tag{5.13}$$

where $J_v(x)$ is the Bessel function of the first kind. Then condition (B) follows from (5.12) and (5.13). From Theorem 3.2 (iv) and Lemma 3.1 we have the following theorem.

THEOREM 5.2. *Assume that the claim size U has the regularly varying distribution* (U) *with index* $\alpha_U > \frac{1}{2}$ *and* $l_U(x) = c_1$ *for some constant* c_1. *Then*

$$\psi(u) \sim \frac{4c_1}{\pi^{3/2} \gamma} u^{1/2 - \alpha_U} \int_0^{+\infty} y^{-1/2} (1+y)^{-\alpha_U} dy.$$

5.3. Ornstein–Uhlenbeck process and $\lambda(x) = x^2 + k$. Let $\{X(t), t \geqslant 0\}$ be a one-parameter Ornstein–Uhlenbeck process with a parameter b such that $X(0) = 0$. That is, $\{X(t), t \geqslant 0\}$ is the diffusion process with the extended generator

$$(Af)(x) = \frac{1}{2} \frac{d^2}{dx^2} f(x) - bx \frac{d}{dx} f(x), \tag{5.14}$$

where $f \in \mathscr{C}^2(\boldsymbol{R})$. The regeneration moments are defined in (5.3). We take $\lambda(x) = x^2 + k$ for $k \geqslant 0$. Hence the intensity process $\{X^2(t) + k, t \geqslant 0\}$ is still the Markov process (see the discussion in Lawrance [27], pp. 225–228). We prove that condition (B) holds, that is

$$E_0^X e^{\delta Z} = E_0^X \exp\{\delta \int_0^T (X^2(t)+k)\,dt\} < +\infty \tag{5.15}$$

for some $\delta > 0$. Then under (U) and (S), by Corollary 3.1, we have

$$\psi(u) \sim C_2 l_U(u) u^{-\alpha_U+1}, \tag{5.16}$$

where C_2 is given in (3.9).

The method of calculating the functional (5.15) consists in changing probability so that the quadratic functional disappears and the remaining problem is to compute m.g.f.'s of some hitting and exit time. We introduce the following exponential change of measure:

$$\frac{dQ_{|\mathcal{F}_t^X}}{dP^X_{|\mathcal{F}_t^X}} = M(t), \tag{5.17}$$

where

$$M(t) = \exp\left\{-\frac{\kappa^2-b^2}{2}\int_0^t X^2(s)\,ds - (\kappa-b)\int_0^t X(s)\,dX(s)\right\} \tag{5.18}$$

$$= \exp\left\{-\frac{\kappa^2-b^2}{2}\int_0^t X^2(s)\,ds - \frac{\kappa-b}{2}(X^2(t)-X^2(0)-t)\right\} \tag{5.19}$$

is an exponential martingale (see Stroock [36], Theorem 4.6, and Rogers and Williams [32], Theorem 27.1). The second equality follows by integration-by-parts for semimartingales. By Stroock [36], Theorem 4.4, and Parthasarathy [31], Theorem 4.2, there exists a unique probability measure Q on $(\mathscr{C}[0, +\infty), \mathcal{F}, \{\mathcal{F}_t^X\}_{t\geq 0})$ fulfilling (5.17). Moreover, by Yor [39], Leblanc et al. [28] and Palmowski and Rolski [30] on the new probability space, the process $\{X(t), t \geq 0\}$ is the Ornstein–Uhlenbeck process with parameter κ. Denote by E_0^Q the expectation with respect to the measure Q. Let $\kappa = \sqrt{b^2-2\delta}$ for $\delta < b^2/2$. Then by the Optional Sampling Theorem we have

$$E_0^X e^{\delta Z} = E_0^X \exp\{\delta \int_0^T X^2(t)\,dt + \delta k T\} \tag{5.20}$$

$$= E_0^Q \exp\{\delta \int_0^T X^2(t)\,dt + \delta k T\} M(T)^{-1}$$

$$= E_0^Q \exp\left\{\frac{\kappa-b}{2}(X^2(T)-X^2(0)-T)+\delta k T\right\} = E_0^Q \exp\left\{\frac{b-\kappa+2\delta k}{2}T\right\}.$$

Let $\hat{\delta} = (b-\kappa+2\delta k)/2$. Note that the following monotone convergence holds:

(5.21) $$\hat{\delta} \to 0^+ \quad \text{as } \delta \to 0^+.$$

Thus it suffices to find $\hat{\delta} > 0$ such that

(5.22) $$E_0^Q \exp\{\hat{\delta} T\} < +\infty.$$

From the Markov property and the symmetry of the Ornstein–Uhlenbeck process we have

(5.23) $$E_0^Q \exp\{\hat{\delta} T\} = E_0^Q \exp\{\hat{\delta} S_0\} E_1^Q \exp\{\hat{\delta} \hat{T}\},$$

where S_0 is exit time from the interval $[-1, 1]$ and $\hat{T} = \inf\{t \geqslant 0\colon X(t) = 0$ and $X(0) = 1\}$. Note that now the parameters of the process $\{X(t), t \geqslant 0\}$ under the new probability measure Q depend on δ, and hence also on $\hat{\delta}$. For this case we state few lemmas. Firstly, we generalize Lemma 5.1 in the following way.

LEMMA 5.2. *Consider a family of diffusion processes $\{X_w(t), t \geqslant 0\}$ parametrized by $w \geqslant 0$ starting at $X(0) = x_w$. If τ_w is an exit time by a diffusion $\{X_w(t), t \geqslant 0\}$ from a compact set D and $E_{x_w}^{X_w} \tau_w \leqslant M$ for all $0 < w \leqslant w_0 < M^{-1}$, then*

$$E_{x_w}^{X_w} \exp\{w\tau_w\} \leqslant 1 + \frac{w_0 M}{1 - w_0 M} \quad \text{for } 0 < w \leqslant w_0.$$

Remark 5.1. Assume that $\{X_w(t), t \geqslant 0\}$ has the following extended generator:

$$(A_w f)(x) = \frac{1}{2} a_w(x) \frac{d^2}{dx^2} f(x) + b_w(x) \frac{d}{dx} f(x)$$

for $f \in \mathscr{C}^2(\boldsymbol{R})$, where functions $a_w(x)$ and $b_w(x)$ fulfill (5.1) and (5.2). If there exists $w_0 > 0$ such that

$$\inf_{w \leqslant w_0} \inf_{x \in D} a_w(x) > 0 \quad \text{and} \quad \sup_{w \leqslant w_0} \sup_{x \in D} |b(x)| \leqslant B < +\infty$$

for some constant B, then by Lemma 5.2 and Wentzell [38], p. 258, $E_{x_w}^{X_w} \exp\{w\tau_w\}$ is uniformly bounded for $0 < w \leqslant w_0$.

LEMMA 5.3. *Let $\{X_w(t), t \geqslant 0\}$ be the family of diffusion processes parametrized by w starting at $X(0) = x_w$ and let*

$$H_z^w = \inf\{t \geqslant 0\colon X_w(t) = z\}$$

be a hitting time. If there exists $w_0 > 0$ such that $E_{x_w}^{X_w} H_z^w \leqslant M$ for all $0 < w \leqslant w_0 < M^{-1}$ and some M, then $E_{x_w}^{X_w} \exp\{w H_z^w\}$ is also uniformly bounded for $0 < w \leqslant w_0$.

Proof. Without loss of generality we can assume that $x_w > z$ for $0 < w \leqslant w_0$. By the Monotone Convergence Theorem,

$$E_{x_w}^{X_w} \exp\{wH_z^w\} = \lim_{R \to +\infty} E_{x_w}^{X_w} \exp\{wH_z^w \wedge T^w(R)\},$$

where

$$T^w(R) = \inf\{t \geqslant 0:\ X_w(t) = R\}.$$

Note that $E_{x_w}^{X_w} H_z^w \wedge T^w(R) \leqslant E_{x_w}^{X_w} H_z^w \leqslant M$ for all $0 < w \leqslant w_0$. Thus, by Lemma 5.2,

$$E_{x_w}^{X_w} \exp\{wH_z^w\} = \lim_{R \to +\infty} E_{x_w}^{X_w} \exp\{wH_z^w \wedge T^w(R)\} \leqslant 1 + \frac{Mw_0}{1 - w_0 M} < +\infty.$$

By Remark 5.1 and (5.21) there exists $\delta_0 > 0$ such that for all $0 < \delta \leqslant \delta_0$

$$E_0^Q \exp\{\hat{\delta S_0}\} < +\infty. \tag{5.24}$$

Moreover, if there exists $\delta_0 > 0$ such that

$$E_1^Q \hat{T} < M$$

for given M and all $0 < \delta \leqslant \delta_0$, then by Lemma 5.3 and (5.21), (5.23), (5.24) the condition (5.22) holds. We calculate $E_1^Q \hat{T}$ using the Laplace transform method. Denote by $D_{-\mu}(x)$ a parabolic cylinder function given by

$$\begin{aligned}(5.25)\quad D_{-\mu}(x) &= \exp\{-x^2/4\} 2^{-\mu/2} \sqrt{\pi} \\ &\times \left\{ \frac{1}{\Gamma((\mu+1)/2)} \left(1 + \sum_{k=1}^{+\infty} \frac{\mu(\mu+2)\ldots(\mu+2k-2)}{3\cdot 5\cdot\ldots\cdot(2k-1)k!} \left(\frac{x^2}{2}\right)^k\right)\right. \\ &\left. - \frac{x\sqrt{2}}{\Gamma(\mu/2)} \left(1 + \sum_{k=1}^{+\infty} \frac{(\mu+1)(\mu+3)\ldots(\mu+2k-1)}{3\cdot 5\cdot\ldots\cdot(2k+1)k!} \left(\frac{x^2}{2}\right)^k\right)\right\}.\end{aligned}$$

Moreover, let

$$s_1(x) = \sum_{k=2}^{+\infty} \frac{2\cdot\ldots\cdot(2k-2)}{3\cdot 5\cdot\ldots\cdot(2k-1)k!} \left(\frac{x^2}{2}\right)^k + \frac{x^2}{2} \tag{5.26}$$

and

$$s_2(x) = \sum_{k=1}^{+\infty} \frac{1}{(2k+1)k!} \left(\frac{x^2}{2}\right)^k. \tag{5.27}$$

From Borodin and Salminen [12], p. 429, we have the following lemma.

Lemma 5.4. *Let $\{X(t), t \geqslant 0\}$ be the Ornstein–Uhlenbeck process with the extended generator* (5.14) *and let*

$$H_z = \inf\{t \geqslant 0:\ X(t) = z\}.$$

Then

$$L^{H_z}(s) = E_x^X \exp\{-sH_z\} = \begin{cases} \dfrac{\exp\{(x^2 b)/2\} D_{-s/b}(-\sqrt{2b}x)}{\exp\{(z^2 b)/2\} D_{-s/b}(-\sqrt{2b}z)} & \text{for } x \leqslant z, \\ \dfrac{\exp\{(x^2 b)/2\} D_{-s/b}(\sqrt{2b}x)}{\exp\{(z^2 b)/2\} D_{-s/b}(\sqrt{2b}z)} & \text{for } z \leqslant x \end{cases}$$

and

$$E_x^X H_z = \begin{cases} b^{-1}\big[(s_1(z\sqrt{2b}) - s_1(x\sqrt{2b})) + \sqrt{b\pi}(z-x) \\ \qquad + \sqrt{b\pi}(zs_2(z\sqrt{2b}) - xs_2(x\sqrt{2b}))\big] & \text{for } x \leqslant z, \\ b^{-1}\big[\sqrt{b\pi}(xs_2(x\sqrt{2b}) - zs_2(z\sqrt{2b})) - (s_1(x\sqrt{2b}) - s_1(z\sqrt{2b})) \\ \qquad + \sqrt{b\pi}(x-z)\big] & \text{for } x \geqslant z. \end{cases}$$

From Lemma 5.4 we obtain

$$\text{(5.28)} \quad E_1^Q \hat{T} = -\frac{d}{ds} L^{\hat{T}}(s)\Big|_{s=0^+} = \frac{1}{\kappa}\big[\sqrt{\kappa\pi}\, s_2(\sqrt{2\kappa}) - s_1(\sqrt{2\kappa}) + \sqrt{\kappa\pi}\big]$$
$$\leqslant \frac{2}{\sqrt{b}}(s_2(\sqrt{2b})+1)$$

for $\delta \leqslant (3b^2)/8$ (then $\kappa \leqslant b/2$). Thus by (5.20) and (5.22) the condition (5.15) is fulfilled, and hence the asymptotics (5.16) holds.

To calculate the constant C_2 in (5.16) explicitly we have to compute $m_{1,T}$ and $m_{1,Z}$. Note that

$$\text{(5.29)} \qquad m_{1,T} = E_0^X T = E_0^X S_0 + E_1^X \hat{T}.$$

By Lemma 5.4 we have

$$\text{(5.30)} \qquad E_1^X \hat{T} = b^{-1}\big[\sqrt{b\pi} s_2(\sqrt{2b}) - s_1(\sqrt{2b}) + \sqrt{b\pi}\big].$$

We calculate $E_0^X S_0$ using the Laplace transform method. Let us put

$$S(\mu, x, y) = \frac{\Gamma(\mu)}{\pi} \exp\{(x^2+y^2)/4\}\big(D_{-\mu}(-x)D_{-\mu}(y) - D_{-\mu}(x)D_{-\mu}(-y)\big).$$

By Borodin and Salminen [12], p. 434, we have the following lemma.

LEMMA 5.5. *Let* $\{X(t),\, t \geqslant 0\}$ *be the Ornstein–Uhlenbeck process with the extended generator* (5.14) *and*

$$H_{a,z} = \inf\{t \geqslant 0\colon\ X(t) \notin (a, z)\}.$$

Then for $a \leqslant x \leqslant z$

$$L^{H_{a,z}}(s) = E_x^X \exp\{-sH_{a,z}\} = \frac{S(s/b, z\sqrt{2b}, x\sqrt{2b}) + S(s/b, x\sqrt{2b}, a\sqrt{2b})}{S(s/b, z\sqrt{2b}, a\sqrt{2b})}$$

and

$$E_x^X H_{a,z} = \frac{A(x, a, z)}{b\big(z(1+s_2(z\sqrt{2b})) - a(1+s_2(a\sqrt{2b}))\big)},$$

where

$$\begin{aligned} A(x, a, z) = {} & z\big(1+s_2(z\sqrt{2b})\big)\big(s_1(a\sqrt{2b}) - s_1(x\sqrt{2b})\big) \\ & + a\big(1+s_2(a\sqrt{2b})\big)\big(s_1(x\sqrt{2b}) - s_1(z\sqrt{2b})\big) \\ & + x\big(1+s_2(x\sqrt{2b})\big)\big(s_1(z\sqrt{2b}) - s_1(a\sqrt{2b})\big). \end{aligned}$$

Lemma 5.5 gives

$$m_{1,S_0} = E_0^X S_0 = \frac{1}{b} s_1(\sqrt{2b}). \tag{5.31}$$

By (5.29)–(5.31) we have

$$m_{1,T} = \frac{\sqrt{\pi}}{\sqrt{b}}[s_2(\sqrt{2b}) + 1]. \tag{5.32}$$

To calculate $m_{1,Z}$ we change the measure by (5.17) using the martingale

$$M(t) = \exp\left\{-\frac{\tilde{\kappa}^2 - b^2}{2}\int_0^t X^2(s)\,ds - \frac{\tilde{\kappa} - b}{2}\big(X^2(t) - X^2(0) - t\big)\right\} \tag{5.33}$$

for $\tilde{\kappa} = \sqrt{b^2 + 2s}$. Then we get

$$\begin{aligned} L^Z(s) &= E_0^X \exp\{-s\int_0^T (X^2(t) + k)\,dt\} \\ &= E_0^Q \exp\{-\hat{s}T\} = E_0^Q \exp\{-\hat{s}S_0\} E_1^Q \exp\{-\hat{s}\hat{T}\}, \end{aligned}$$

where $\hat{s} = (\kappa - b + 2sk)/2$ and under the probability measure Q the process $\{X(t), t \geqslant 0\}$ is the Ornstein–Uhlenbeck process with parameter $\tilde{\kappa}$. From Lemmas 5.5 and 5.4 we obtain

$$E_0^Q \exp\{-\hat{s}S_0\} = \frac{S(\hat{s}/\tilde{\kappa}, \sqrt{2\tilde{\kappa}}, 0) + S(\hat{s}/\tilde{\kappa}, 0, -\sqrt{2\tilde{\kappa}})}{S(\hat{s}/\tilde{\kappa}, \sqrt{2\tilde{\kappa}}, -\sqrt{2\tilde{\kappa}})}$$

and

$$E_1^Q \exp\{-\hat{s}\hat{T}\} = \frac{\exp\{\tilde{\kappa}/2\} D_{-\hat{s}/\tilde{\kappa}}(\sqrt{2\tilde{\kappa}})}{D_{-\hat{s}/\tilde{\kappa}}(0)}.$$

Thus

$$m_{1,Z} = -\frac{d}{ds}L^Z(s)\Big|_{s=0^+} = \frac{\sqrt{\pi}}{\sqrt{b}}(s_2(\sqrt{2b})+1)\left(\frac{1}{2b}+k\right). \tag{5.34}$$

Summarizing, from (5.16) we have the following theorem.

THEOREM 5.3. *Assume that* $\{X^2(t)+k,\ t \geqslant 0\}$ *is the intensity process for* $k \geqslant 0$ *and for the Ornstein–Uhlenbeck process* $\{X(t),\ t \geqslant 0\}$ *with parameter* b *starting at* $X(0) = 0$. *If the claim size* U *has the regularly varying distribution* (U) *and* $m_{1,U} < 2b/(1+2bk)$, *then*

$$\psi(u) \sim \frac{1}{\alpha_U - 1}\frac{1/2b+k}{1-m_{1,U}(1/2b+k)} l_U(u)\, u^{-\alpha_U+1}.$$

5.4. Ornstein–Uhlenbeck process and $\lambda(x) = (x+p)^2$. Let $\{X(t),\ t \geqslant 0\}$ be the Ornstein–Uhlenbeck process with parameter b starting at $X(0) = 0$. We define the regeneration moments by (5.3). We take $\lambda(x) = (x+p)^2$. Hence the intensity process $\{(X(t)+p)^2,\ t \geqslant 0\}$ is non-Markovian. We prove condition (B) as in the previous section. Then, by Corollary 3.1 under conditions (U) and (S), the asymptotics (3.19) hold. We introduce the exponential change of the measure (5.17), where

$$M(t) = \exp\left\{\frac{b^2}{2}\int_0^t X^2(w)\,dw + \frac{b}{2}(X^2(t) - X^2(0) - t)\right\}. \tag{5.35}$$

By the Cameron–Martin–Girsanov Theorem under the new probability measure Q the process $\{X(t),\ t \geqslant 0\}$ is the Brownian motion. Hence

$$E_0^X e^{\delta Z} = E_0^Q M^{-1}(T) \exp\{\delta \int_0^T (X(t)+p)^2\,dt\}$$

$$= E_0^Q \exp\left\{\frac{-\kappa^2}{2}\int_0^T (X(t)-\tilde{p})^2\,dt + \left(\frac{\kappa^2}{2}\tilde{p}^2 + \delta p^2 + \frac{b}{2}\right)T\right\},$$

where $\kappa = \sqrt{b^2 - 2\delta}$ and $\tilde{p} = (2p\delta)/\kappa^2$. Let

$$\tilde{S}_0 = \inf\{t \geqslant 0\colon |X(t)+\tilde{p}| = 1\}, \qquad \tilde{T} = \inf\{t \geqslant \tilde{S}_0\colon X(t) = -\tilde{p}\}.$$

Then

$$E_0^X e^{\delta Z} = E_{-\tilde{p}}^Q \exp\left\{\frac{\kappa^2}{2}\int_0^{\tilde{T}} X^2(t)\,dt + \left(\frac{\kappa^2}{2}\tilde{p}^2 + \delta p^2 + \frac{b}{2}\right)\tilde{T}\right\}.$$

We change again the measure in the following way:

$$\frac{d\tilde{Q}_{|\mathcal{F}_t^X}}{dQ^X_{|\mathcal{F}_t^X}} = \tilde{M}(t),$$

where

$$\tilde{M}(t) = \exp\left\{-\frac{\kappa^2}{2}\int_0^t X^2(w)\,dw - \frac{\kappa}{2}\left(X^2(t) - X^2(0) - t\right)\right\}.$$

Then under the probability measure $\tilde{Q}$ the process $\{X(t),\ t \geqslant 0\}$ is the Ornstein–Uhlenbeck process with parameter κ. We have

$$(5.36)\quad E_0^X e^{\delta Z} = E_{-\tilde{p}}^{\tilde{Q}} \exp\left\{\left(-\frac{\kappa}{2} + \frac{\kappa^2}{2}\tilde{p}^2 + \delta p^2 + \frac{b}{2}\right)\tilde{T}\right\} = E_{-\tilde{p}}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}\},$$

where

$$\tilde{\delta} = b/2 - \kappa/2 + p\delta\tilde{p} + \delta p^2 > 0.$$

Note that

$$(5.37)\qquad \tilde{\delta} \to 0^+ \quad \text{as } \delta \to 0^+.$$

Let us put

$$\tilde{T}_0 = \inf\{t \geqslant 0:\ X(t) = -\tilde{p}\}.$$

By the Markov property

$$\begin{aligned} E_{-\tilde{p}}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}\} &= E_{-\tilde{p}}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{S}_0\}\,[E_{-\tilde{p}-1}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}_0\}\cdot\tilde{Q}(X(\tilde{S}_0) = -\tilde{p}-1) \\ &\quad + E_{-\tilde{p}+1}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}_0\}\cdot\tilde{Q}(X(\tilde{S}_0) = -\tilde{p}+1)] \\ &\leqslant E_{-\tilde{p}}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{S}_0\}\,(E_{-\tilde{p}-1}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}_0\} + E_{-\tilde{p}+1}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{T}_0\}). \end{aligned}$$

By Remark 5.1 and (5.37) there exists $\delta_0 > 0$ such that $E_{-\tilde{p}}^{\tilde{Q}} \exp\{\tilde{\delta}\tilde{S}_0\}$ is uniformly bounded for all $0 < \delta \leqslant \delta_0$. Thus to prove (B) it suffices by Lemma 5.3 and (5.37) to find $\delta_0 > 0$ such that $E_{-\tilde{p}-1}^{\tilde{Q}}\tilde{T}_0$ and $E_{-\tilde{p}+1}^{\tilde{Q}}\tilde{T}_0$ are uniformly bounded for all $0 < \delta \leqslant \delta_0$. Lemma 5.4 gives

$$\begin{aligned} E_{-\tilde{p}-1}^{\tilde{Q}}\tilde{T}_0 &= \kappa^{-1}\left[s_1(\tilde{p}\sqrt{2\kappa}) + \sqrt{\kappa\pi} + \sqrt{\kappa\pi}(\tilde{p}+1)s_2(\sqrt{\kappa\pi}(\tilde{p}+1))\right] \\ &\leqslant 2b^{-1}\left[s_1(\sqrt{2b}) + \sqrt{b\pi} + 2\sqrt{b\pi}\,s_2(2\sqrt{2b})\right], \\ E_{-\tilde{p}+1}^{\tilde{Q}}\tilde{T}_0 &\leqslant 2b^{-1}\left[s_1(\sqrt{2b}) + \sqrt{b\pi} + 2\sqrt{b\pi}\,s_2(\sqrt{2b})\right] \end{aligned}$$

for all $\delta < (3b^2)/8 \wedge b^2/(8|p|)$ (then $|\tilde{p}| \leqslant 1$ and $b \geqslant \kappa \geqslant b/2$). We now calculate $m_{1,Z}$ needed for obtaining the constant C_2 in (3.19) explicitly. The constant $m_{1,T}$ is given in (5.32). Note that the Laplace transform of Z equals

$$\begin{aligned} L^Z(s) = E_0^X e^{-sZ} &= E_{-\tilde{p}_0}^{\tilde{Q}} \exp\{-\tilde{s}\tilde{T}\} \\ &= E_{-\tilde{p}_0}^{\tilde{Q}} \exp\{-\tilde{s}\tilde{S}_0\}\left(E_{-\tilde{p}_0-1}^{\tilde{Q}} \exp\{-\tilde{s}\tilde{T}_0\}\cdot\tilde{Q}(X(\tilde{S}_0) = -\tilde{p}_0-1)\right. \\ &\quad \left. + E_{-\tilde{p}_0+1}^{\tilde{Q}} \exp\{-\tilde{s}\tilde{T}_0\}\cdot\tilde{Q}(X(\tilde{S}_0) = -\tilde{p}_0+1)\right), \end{aligned}$$

where under the measure $\tilde{Q}$ the process $\{X(t),\, t \geqslant 0\}$ is the Ornstein–Uhlenbeck process with parameter $\tilde{\kappa} = \sqrt{b^2+2s}$. Moreover, $\tilde{p}_0 = -(2ps)/\tilde{\kappa}^2$ and

$$\tilde{s} = \tilde{\kappa}/2 - b/2 - ps\tilde{p}_0 + sp^2 > 0$$

for sufficiently small s. From Lemmas 5.5 and 5.4 we have

$$E^{\tilde{Q}}_{-\tilde{p}_0-1} \exp\{-\tilde{s}\tilde{T}_0\} = \frac{\exp\{\tilde{p}_0^2\tilde{\kappa}/2\}\, D_{-\tilde{s}/\tilde{\kappa}}(-\tilde{p}_0\sqrt{2\tilde{\kappa}})}{\exp\{(\tilde{p}_0+1)^2\tilde{\kappa}/2\}\, D_{-\tilde{s}/\tilde{\kappa}}(-(\tilde{p}_0+1)\sqrt{2\tilde{\kappa}})},$$

$$E^{\tilde{Q}}_{-\tilde{p}_0+1} \exp\{-\tilde{s}\tilde{T}_0\} = \frac{\exp\{\tilde{p}_0^2\tilde{\kappa}/2\}\, D_{-\tilde{s}/\tilde{\kappa}}(-\tilde{p}_0\sqrt{2\tilde{\kappa}})}{\exp\{(1-\tilde{p}_0)^2\tilde{\kappa}/2\}\, D_{-\tilde{s}/\tilde{\kappa}}(-(1-\tilde{p}_0)\sqrt{2\tilde{\kappa}})},$$

$$E^{\tilde{Q}}_{-\tilde{p}_0} \exp\{-\tilde{s}\tilde{S}_0\} = \frac{S(\tilde{s}/\tilde{\kappa},\, (1-\tilde{p}_0)\sqrt{2\tilde{\kappa}},\, -\tilde{p}_0\sqrt{2\tilde{\kappa}}) + S(\tilde{s}/\tilde{\kappa},\, -\tilde{p}_0\sqrt{2\tilde{\kappa}},\, -(\tilde{p}_0+1)\sqrt{2\tilde{\kappa}})}{S(\tilde{s}/\tilde{\kappa},\, (1-\tilde{p}_0)\sqrt{2\tilde{\kappa}},\, -(\tilde{p}_0+1)\sqrt{2\tilde{\kappa}})}.$$

Let us put

$$\operatorname{Erfi}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\{-v^2\}\, dv, \qquad \operatorname{Erfi}(-x) = -\operatorname{Erfi}(x),$$

$$\operatorname{Erfi} d(x, y) = \operatorname{Erfi}\left(\frac{x}{\sqrt{2}}\right) - \operatorname{Erfi}\left(\frac{y}{\sqrt{2}}\right)$$

and

$$C(v, x, y) = \frac{\Gamma(v+1)}{\pi} \exp\{(x^2+y^2)/4\}\big(D_{-v-1}(-x)\, D_{-v}(y) + D_{-v-1}(x)\, D_{-v}(-y)\big).$$

By Borodin and Salminen [12] the following holds:

$$\tilde{Q}(X(\tilde{S}_0) = -\tilde{p}_0 - 1) = 1 - \tilde{Q}(X(\tilde{S}_0) = -\tilde{p}_0 + 1) = \frac{\operatorname{Erfi} d((1-\tilde{p}_0)\sqrt{2\tilde{\kappa}},\, -\tilde{p}_0\sqrt{2\tilde{\kappa}})}{\operatorname{Erfi} d((1-\tilde{p}_0)\sqrt{2\tilde{\kappa}},\, -(\tilde{p}_0+1)\sqrt{2\tilde{\kappa}})}.$$

Note that

$$\frac{d}{dx} D_{-v}(x) = -\frac{x}{2} D_{-v}(x) - v D_{-v-1}(x),$$

$$\frac{d}{ds} D_{-s}(x) = \exp\{-x^2/4\}\left[s_2(x) - \frac{x}{\sqrt{2\pi}}(1 + s_1(x))\right]$$

and

$$\frac{\partial}{\partial x} S(v, x, y) = C(v, x, y), \qquad \frac{\partial}{\partial y} S(v, x, y) = -C(v, x, y).$$

Hence

$$(5.38)\quad m_{1,Z} = -\frac{d}{ds}(E_0^X e^{-sZ})|_{s=0^+} = -\frac{d}{ds}(E^{\tilde{Q}}_{-\tilde{p}_0} \exp\{-\tilde{s}\tilde{S}_0\})|_{s=0^+}$$

$$-\frac{1}{2}\left(\frac{d}{ds}(E^{\tilde{Q}}_{-\tilde{p}_0-1} \exp\{-\tilde{s}\tilde{T}_0\})|_{s=0^+} + \frac{d}{ds}(E^{\tilde{Q}}_{-\tilde{p}_0+1} \exp\{-\tilde{s}\tilde{T}_0\})|_{s=0^+}\right)$$

$$= \frac{\sqrt{2}}{4\sqrt{\pi}\,\mathrm{Erfi}(\sqrt{b})^2}\left\{(e^b+1)\left[2s_1(\sqrt{2b})\,\mathrm{Erfi}(\sqrt{b})\left(\frac{1}{2b^2}+\frac{p^2}{b}\right)+\frac{2}{\sqrt{\pi}}\frac{1}{b^{3/2}}e^b\right]\right.$$

$$\left.-\frac{2\sqrt{2}}{b^{3/2}}e^b\,\mathrm{Erfi}(\sqrt{b})\right\}+\frac{1}{b}+\left(\frac{1}{2b^2}+\frac{p^2}{b}\right)\left[s_2(\sqrt{2b})+\frac{\sqrt{b}}{\sqrt{\pi}}(1+s_1(\sqrt{2b}))\right].$$

Summarizing we have the following theorem.

THEOREM 5.4. *Assume that* $\{(X(t)+p)^2,\ t \geqslant 0\}$ *is the intensity process for* $p \in \boldsymbol{R}$ *and for the Ornstein–Uhlenbeck process* $\{X(t),\ t \geqslant 0\}$ *with parameter* b *starting at* $X(0) = 0$. *If the claim size* U *has the regularly varying distribution* (U) *and* (S) *holds, then*

$$\psi(u) \sim \frac{1}{\alpha_U - 1} m_{1,Z} \frac{1}{m_{1,T} - m_{1,U} m_{1,Z}} l_U(u) u^{-\alpha_U+1},$$

where $m_{1,T}$ *and* $m_{1,Z}$ *are given in* (5.32) *and* (5.38), *respectively.*

5.5. Splitting Brownian bridges and $\lambda(x) = |x|$. We construct the governing process $\{X(t),\ t \geqslant 0\}$ by splitting independent Brownian bridges defined on the interval $[n, n+1]$ $(n \in N)$. That is, let $\{Z(t),\ t \in [0, 1]\}$ be a Brownian bridge (see Karatzas and Shreve [24], p. 358, for construction of the Brownian bridge). Define the sequence $\{Z_n(t)\}$, $n = 1, 2, \ldots$, of independent copies of $Z(t)$. Then $X(t) = Z_n(n+t)$ if $t \in [n, n+1]$. Hence $T_n = n$ are moments of regeneration and $T = 1$. Let $\lambda(x) = |x|$. Thus on each interval $[n, n+1]$ the intensity process is the reflecting Brownian bridge. By Karatzas and Shreve [24], p. 360,

$$(5.39)\qquad Z(t) \stackrel{D}{=} B(t) - tB(1).$$

Thus

$$(5.40)\qquad Z = \int_0^1 |X(t)|\,dt = \int_0^1 |Z(t)|\,dt \stackrel{D}{=} \int_0^1 |B(t) - tB(1)|\,dt.$$

Note that

$$(5.41)\qquad Z \leqslant \int_0^1 \Big(\sup_{0 \leqslant t \leqslant 1} |B(t)| + t|B(1)|\Big)\,dt \leqslant 2 \sup_{0 \leqslant t \leqslant 1} |B(t)|.$$

Hence

$$P(Z > x) \leqslant P\Big(\sup_{0\leqslant t\leqslant 1} |B(t)| > \frac{x}{2}\Big) \leqslant 2P\Big(\sup_{t\in[0,1]} B(t) > \frac{x}{2}\Big)$$
$$= 4\frac{1}{\sqrt{2\pi}}\int_{x/2}^{+\infty} \exp\Big\{-\frac{y^2}{2}\Big\}dy$$

by Adler [2] and Karatzas and Shreve [24], p. 96. Thus condition (B) is fulfilled. Moreover, by the Fubbini Theorem,

$$m_{1,Z} = \int_0^1 (E_0^B|B(t)-tB(1)|)\,dt$$
$$= \int_0^1 \frac{1}{2\pi\sqrt{1-t}}\int_{-\infty}^{+\infty} \exp\{-x^2/2\}\int_{-\infty}^{+\infty} |x-ty|\exp\Big\{-\frac{(x-y)^2}{2(1-t)}\Big\}dy\,dx\,dt$$
$$= \frac{\sqrt{2}}{\sqrt{\pi}}\int_0^1 \frac{1}{\sqrt{t^2-t+1}}(t^2+t\sqrt{1-t})\,dt = \frac{\sqrt{2}}{\sqrt{\pi}}\Big[\frac{1}{2}-\frac{1}{8}\log(3)+\int_0^1 \frac{t\sqrt{1-t}}{\sqrt{t^2-t+1}}dt\Big].$$

Taking the substitution $y^2 := 1-t$ we get

$$(5.42)\quad m_{1,Z} = \frac{\sqrt{2}}{\sqrt{\pi}}\Big[\frac{1}{2}-\frac{1}{8}\log(3)-2\int_0^1\sqrt{y^4-y^2+1}\,dy+2\int_0^1\frac{dy}{\sqrt{y^4-y^2+1}}\Big]$$
$$= \frac{\sqrt{2}}{\sqrt{\pi}}\Big[\mathrm{Elliptic}\,K\Big(\frac{\sqrt{3}}{2}\Big)-\frac{2\sqrt{2}}{3}\mathrm{Elliptic}\,F\Big(\frac{2}{3}\sqrt{2}, \frac{1}{4}\sqrt{10}\Big)-\frac{1}{6}-\frac{1}{8}\log(3)\Big]$$
$$\simeq 0.79788,$$

where

$$\mathrm{Elliptic}\,F(z, k) = \int_0^z \frac{1}{\sqrt{1-y^2}\sqrt{1-k^2y^2}}dy$$

is the incomplete integral of the first kind and $\mathrm{Elliptic}\,K(k) = \mathrm{Elliptic}\,F(1, k)$ is the complete elliptic integral of the first kind (see Abramowitz and Stegun [1], Chapter 17). By Corollary 3.1 we have the following theorem.

THEOREM 5.5. *Assume that the intensity process is constructed by splitting reflecting Brownian bridges. If the claim size U has the regularly varying distribution* (U) *with* $\alpha_U > 1$ *and* $m_{1,Z}m_{1,U} < 1$, *then*

$$\psi(u) \sim \frac{1}{\alpha_U-1}m_{1,Z}\frac{1}{1-m_{1,U}m_{1,Z}}l_U(u)\,u^{-\alpha_U+1},$$

where $m_{1,Z}$ *is given in* (5.42).

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